Math 115

Spring 2018

Lecture 26

$? \ a^2 + b^2 = c^2 \ ?$
$?$
$y = mx + b \quad d = rt$

Last Chapter: Rational Expressions

$$\frac{Polynomial}{Polynomial} \qquad \frac{x^2-4}{x^2+3x+2} \ , \ \frac{6x+10}{10x+8} \ , \ \frac{x^2-2x-48}{x^2-2x-35}$$

How to reduce/Simplify them:

① Factor numerator and denominator completely.

② Cross out any common factor.

Simplify

$$\frac{6x+10}{8x-14} = \frac{\cancel{2}(3x+5)}{\cancel{2}(4x-7)} = \boxed{\frac{3x+5}{4x-7}}$$

$$\frac{5x^2-15x}{10x^2+25x} = \frac{\cancel{5x}(x-3)}{\cancel{5x}(2x+5)} = \boxed{\frac{x-3}{2x+5}}$$

Simplify

① $\dfrac{x^2-4}{x^2+3x+2}$

$= \dfrac{(x+2)(x-2)}{(x+1)(x+2)}$

$= \boxed{\dfrac{x-2}{x+1}}$

② $\dfrac{x^2-2x-48}{x^2-36}$

$= \dfrac{(x-8)(x+6)}{(x+6)(x-6)}$

$= \boxed{\dfrac{x-8}{x-6}}$

③ $\dfrac{2x^2+3x-5}{3x^2+2x-5}$

$= \dfrac{(2x+5)(x-1)}{(3x+5)(x-1)}$

$= \boxed{\dfrac{2x+5}{3x+5}}$

④ $\dfrac{x^3+5x^2+25x}{x^3-125}$

$= \dfrac{x(x^2+5x+25)}{x^3-5^3}$

$= \dfrac{x(x^2+5x+25)}{(x-5)(x^2+5x+25)}$

$= \boxed{\dfrac{x}{x-5}}$

Excluded Values: Any value that makes the rational expression undefined is called excluded value.

$$\frac{\text{Polynomial}}{\text{Polynomial}}$$

Deno. = 0

To find excluded Values:
① Deno. = 0
② Solve

find all excluded Values

$$\frac{x+1}{x-5} \longrightarrow x-5=0$$

$\boxed{x=5}$ $\boxed{\text{E.V.}:5}$

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find all excluded Values:

① $\dfrac{2x-7}{3x+5}$

$3x+5=0$

$3x=-5$

$\boxed{x=\dfrac{-5}{3}}$ $\boxed{\text{E.V.}:\dfrac{-5}{3}}$

② $\dfrac{x-7}{x^2-2x-35}$

$x^2-2x-35=0$

$(x-7)(x+5)=0$

By Z.P.R.

$x-7=0$ $x+5=0$

$\boxed{x=7}$ $\boxed{x=-5}$

E.V. $-5 \& 7$

find all excluded values:

① $\dfrac{2x-9}{x^2-36}$

Deno. $=0$, Solve

$x^2-36=0$

$(x+6)(x-6)=0$

by Z.F.R.

$x+6=0$ $\qquad$ $x-6=0$

$\boxed{x=-6}$ $\qquad$ $\boxed{x=6}$

E.V.: ±6

E.V. -1 & $\dfrac{7}{3}$

② $\dfrac{3x^2+7x}{3x^2-4x-7}$

Deno. $=0$, Solve

$3x^2-4x-7=0$

$ax^2+bx+c=0$

$a=3 \qquad b=-4 \qquad c=-7$

$b^2-4ac=(-4)^2-4(3)(-7)=$

$=16+84=100$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\dfrac{-(-4)\pm\sqrt{100}}{2(3)}$

$=\dfrac{4\pm10}{6}$

$x=\dfrac{4+10}{6}=\boxed{\dfrac{7}{3}}$

$x=\dfrac{4-10}{6}=\boxed{-1}$

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find all excluded values:

$\dfrac{x^2-6x+1}{5x^2+2x-3}$

$b^2-4ac=$

$(2)^2-4(5)(-3)=64$

$x=\dfrac{-2\pm\sqrt{64}}{2(5)}=\dfrac{-2\pm8}{10}$

E.V.: -1 & $\dfrac{3}{5}$

Deno. $=0$, Solve

$5x^2+2x-3=0$

$ax^2+bx+c=0$

$a=5 \qquad b=2 \qquad c=-3$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\dfrac{-2+8}{10}=\dfrac{6}{10}=\boxed{\dfrac{3}{5}}$

$x=\dfrac{-2-8}{10}=\dfrac{-10}{10}=\boxed{-1}$

How to multiply rational expressions:

① factor everything completely

② Cross-out common factors vertically or diagonally.

③ write the remaining factors in product form.

Multiply

$$\frac{x^2-4}{x^2+7x+10} \cdot \frac{x^2-25}{x+7} = \frac{(x+2)(x-2)}{(x+2)(x+5)} \cdot \frac{(x+5)(x-5)}{x+7}$$

$$= \boxed{\frac{(x-2)(x-5)}{x+7}}$$

Multiply:

$$\frac{x^2+7x+12}{x^2-16} \cdot \frac{x^2+2x-24}{x^2+3x}$$

$$= \frac{(x+4)(x+3)}{(x+4)(x-4)} \cdot \frac{(x-4)(x+6)}{x(x+3)}$$

$$= \boxed{\frac{x+6}{x}}$$

Multiply

$$\frac{m^2+3m+2}{m^2+5m+4} \cdot \frac{m^2+10m+24}{m^2+5m+6} \cdot \frac{m^2-1}{m^2+1}$$

$$= \frac{(m+2)(m+1)}{(m+1)(m+4)} \cdot \frac{(m+4)(m+6)}{(m+2)(m+3)} \cdot \frac{(m+1)(m-1)}{m^2+1}$$

$$= \boxed{\frac{(m+1)(m+6)(m-1)}{(m+3)(m^2+1)}}$$

Multiply:

$$\frac{2m^2-5m-12}{m^2-10m+24} \cdot \frac{m^2-9m+18}{4m^2-9} \quad \to (2m)^2-3^2$$

$$= \frac{(2m+3)(m-4)}{(m-4)(m-6)} \cdot \frac{(m-6)(m-3)}{(2m-3)(2m+3)}$$

$$= \boxed{\frac{m-3}{2m-3}}$$

$$\frac{m-3}{2m-3} \quad \text{wrong}$$

How to divide rational expressions:

① Change $\div$ to $\cdot$ with the reciprocal

② factor everything Completely

③ Simplify.

Divide

$$\frac{3x-27}{4} \div \boxed{\frac{2x-18}{12}} = \frac{3x-27}{4} \cdot \frac{12}{2x-18}$$

$$= \frac{3(x-9)}{4_1} \cdot \frac{\cancel{12}^3}{2(x-9)}$$

$$= \boxed{\frac{9}{2}}$$

Divide:

$$\frac{x^2-4}{8x-16} \div \frac{x+2}{8} = \frac{(x+2)(x-2)}{8(x-2)} \cdot \frac{8}{x+2}$$

$$= \frac{8}{8} = \boxed{1}$$

Simplify:

$$\frac{x^2+2x-3}{x^2-3x+2} \div \frac{x^2+4x+3}{x^2+2x-8}$$

$$= \frac{x^2+2x-3}{x^2-3x+2} \cdot \frac{x^2+2x-8}{x^2+4x+3}$$

$$= \frac{(x+3)(x-1)}{(x-2)(x-1)} \cdot \frac{(x+4)(x-2)}{(x+3)(x+1)} = \boxed{\frac{x+4}{x+1}}$$

Solve by Zero-Product rule:

$$(3x-5)(4x+7)(7x-2)=0$$

$3x-5=0$ or $4x+7=0$ or $7x-2=0$

$x=\dfrac{5}{3}$ $x=\dfrac{-7}{4}$ $x=\dfrac{2}{7}$

$$\left\{\dfrac{-7}{4}, \dfrac{2}{7}, \dfrac{5}{3}\right\}$$

Solve by Factoring Method:

$x^2-2x=80$ ⎫ $2x^2+6=13x$

$x^2-2x-80=0$ $2x^2+6-13x=0$

$(x-10)(x+8)=0$ $2x^2-13x+6=0$

By Z.P.R. $(2x-1)(x-6)=0$

$x-10=0$ or $x+8=0$ By Z.P.R.

$\boxed{x=10}$ $\boxed{x=-8}$ $2x-1=0$ or $x-6=0$

$\{-8,10\}$ $x=\dfrac{1}{2}$ $x=6$

$\left\{\dfrac{1}{2},6\right\}$

FOIL, Simplify, Solve by factoring method:

$(2x+3)(3x-1) = 35$

$6x^2 - 2x + 9x - 3 - 35 = 0$

$6x^2 + 7x - 38 = 0$

$P = -228$
$S = 7$ -228

-1, 228
-2, 114
-3, 76
-4, 57
-6, 38
$\boxed{-12, 19}$

$6x^2 - 12x + 19x - 38$

$6x(x-2) + 19(x-2)$

$(x-2)(6x+19) = 0$

$\left\{ \dfrac{-19}{6}, 2 \right\}$

by Z.P.R. $x - 2 = 0$ $6x + 19 = 0$
              $x = 2$ $x = \dfrac{-19}{6}$

FOIL, Simplify, Solve by Q-formula:

$(2x+5)(3x-4) = 18$

$6x^2 - 8x + 15x - 20 - 18 = 0$

$6x^2 + 7x - 38 = 0$ $a = 6, \ b = 7, \ c = -38$

$ax^2 + bx + c = 0$ $b^2 - 4ac = (7)^2 - 4(6)(-38)$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= 961$

$x = \dfrac{-7 \pm \sqrt{961}}{12}$
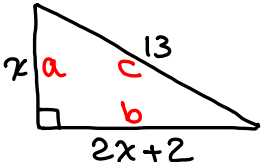
$x = \dfrac{-7 \pm 31}{12}$ $x = \dfrac{-7+31}{12} = \dfrac{24}{12}$ $x = \dfrac{-7-31}{12} = \dfrac{-38}{12} = \boxed{\dfrac{-19}{6}}$

$= \boxed{2}$

$\left\{ \dfrac{-19}{6}, 2 \right\}$

find x

Right Triangle
Pythagorean thrm

$$a^2 + b^2 = c^2$$

$$x^2 + (2x+2)^2 = 13^2$$

$$5^2 + 12^2 = 13^2$$

$$x^2 + (2x+2)(2x+2) = 169$$

$$x^2 + 4x^2 + 8x + 4 - 169 = 0$$

$$5x^2 + 8x - 165 = 0$$

use Q-formula to solve.

$a=5 \qquad b=8 \qquad c=-165$

$$b^2 - 4ac = (8)^2 - 4(5)(-165) = 3364$$

$$x = \frac{-8 \pm \sqrt{3364}}{2(5)} = \frac{-8 \pm 58}{10}$$

$$x = \frac{-8 + 58}{10} = \boxed{5}$$

$$x = \frac{-8 - 58}{10} = -\#$$

Next Project:

Polynomial Equations

1 - 40

Use 4-Part Page

one Side only

Due Thursday

John invested some money in 2% simple interest account, and $500 more than that in 5% simple interest account.
After 4 years, he earned $310 in simple interest. How much per account?

Accounts	P	r	t	I
First Acct	x	.02	4	$x(.02)(4)$
Second Acct	$x+500$	.05	4	$(x+500)(.05)(4)$

$.08x + .2(x+500) = 310$

$.08x + .2x + 100 = 310$

$.28x = 210$

$x = \dfrac{210}{.28}$ ⬚ $x = 750$

$750 @ 2\% rate
&
$1250 @ 5\% rate

Jake had $3000. He invested some in 5.5% account, and the rest of it, in 7.5% account.
After 1 year, his total earned simple interest was $189. How much per account?

Accts	P	r	t	I
Acct I	x	.055	1	$.055x$
Acct II	$3000-x$	.075	1	$.075(3000-x)$

$.055x + .075(3000-x) = 189$

$.055x + 225 - .075x = 189$

$-.02x = 189 - 225$

$-.02x = -36$

$x = \dfrac{-36}{-.02}$

$x = 1800$

$1800 @ 5.5% account
&
$1200 @ 7.5% account

Mike put some money in simple interest at 3% APR.

He also put $500 less than twice that amount in simple interest at 4% APR.

He earned $105 more interest in one year from the second account. How much per Acct?

Acct	P	R	T	I
Acct I	x	3%	1	$.03x$
Acct II	$2x-500$	4%	1	$.04(2x-500)$

Acct II interest = Acct I interest + 105

$$.04(2x-500) = .03x + 105$$

$$.08x - 20 = .03x + 105$$

$$.08x - .03x = 105 + 20$$

$$.05x = 125$$

$$x = 2500$$

$2500 @ 3% Acct
&
$4500 @ 4% Acct